

1. (a) Find, to 3 significant figures, the value of  $x$  for which  $5^x = 7$ . (2)

- (b) Solve the equation  $5^{2x} - 12(5^x) + 35 = 0$ . (4)  
(Total 6 marks)

2. Solve the equation

$$5^x = 17,$$

giving your answer to 3 significant figures.

(Total 3 marks)

3. Solve

- (a)  $5^x = 8$ , giving your answers to 3 significant figures, (3)

- (b)  $\log_2(x + 1) - \log_2 x = \log_2 7$ . (3)  
(Total 6 marks)

4. Find, giving your answer to 3 significant figures where appropriate, the value of  $x$  for which

- (a)  $3^x = 5$ , (3)

- (b)  $\log_2(2x + 1) - \log_2 x = 2$ . (4)  
(Total 7 marks)

1. (a)  $x = \frac{\log 7}{\log 5}$  or  $x = \log_5 7$  (i.e. correct method up to  $x = \dots$ )  
 1.21 Must be this answer (3 s.f.) A1 2  
 1.21 with no working: A1 (even if it left as  $5^{1.21}$ ).  
 Other answers which round to 1.2 with no working: A0.
- (b)  $(5^x - 7)(5^x - 5)$  Or another variable, e.g.  $(y - 7)(y - 5)$ ,  
 even  $(x - 7)(x - 5)$  M1A1  
 $(5^x = 7 \text{ or } 5^x = 5)$   $x = 1.2$  (awrt) ft from the answer to (a), if used A1ft  
 $x = 1$  (allow 1.0 or 1.00 or 1.000) B1 4

M: Using the correct quadratic equation, attempt to factorise  $(5^x \pm 7)(5^x \pm 5)$ , or attempt quadratic formula.

Allow  $\log_5 7$  or  $\frac{\log 7}{\log 5}$  instead of 1.2 for A1ft.

No marks for simply substituting a decimal answer from (a) into the given equation (perhaps showing that it gives approximately zero).

However, note the following special case:

Showing that  $5^x = 7$  satisfies the given equation, therefore 1.21 is a solution scores 0, 0, 1, 0 (and could score full marks if the  $x = 1$  were also found).

e.g. If  $5^x = 7$ , then  $5^{2x} = 49$ , and  $5^{2x} - 12(5^x) + 35 = 49 - 84 + 35 = 0$ , so one solution is  $x = 1.21$  ('conclusion' must be seen).

To score this special case mark, values substituted into the equation must be exact. Also, the mark would not be scored in the following case:

e.g. If  $5^x = 7$ ,  $5^{2x} - 84 + 35 = 0 \Rightarrow 5^{2x} = 49 \Rightarrow x = 1.21$   
 (Showing no appreciation that  $5^{2x} = (5^x)^2$ )

B1: Do not award this mark if  $x = 1$  clearly follows from wrong working.

[6]

2.  $x \log 5 = \log 17$  or  $x = \log_5 17$   
 $x = \frac{\log 17}{\log 5}$  A1  
 $= 1.76$  A1 3

N.B. It is never possible to award an A mark after giving M0.  
If M0 is given then the marks will be M0 A0 A0.

Acceptable alternatives include

$$x \log 5 = \log 17; x \log_{10} 5 = \log_{10} 17; x \log_e 5 = \log_e 17; x \ln 5 = \ln 17; x = \log_5 17$$

Can be implied by a correct exact expression as shown on the first A1 mark

1<sup>st</sup>

An exact expression for  $x$  that can be evaluated on a calculator.

Acceptable alternatives include

$$x = \frac{\log 17}{\log 5}; x = \frac{\log_{10} 17}{\log_{10} 5}; x = \frac{\log_e 17}{\log_e 5}; x = \frac{\ln 17}{\ln 5}; x = \frac{\log_q 17}{\log_q 5}$$

where  $q$  is a number

This may not be seen (as, for example,  $\log_5 17$  can be worked out directly on many calculators) so this A mark can be implied by the correct final answer or the right answer corrected to or truncated to a greater accuracy than 3 significant figures or 1.8

Alternative:  $x = \frac{\text{a number}}{\text{a number}}$  where this fraction, when worked out

as a decimal rounds to 1.76.

(N.B. remember that this A mark cannot be awarded without the M mark).

If the line for the M mark is missing but this line is seen (with or without the  $x =$ ) and is correct the method can be assumed and

1<sup>st</sup> A1 given.

1<sup>st</sup> A1

1.76 cao

2<sup>nd</sup> A1

N.B.  $\sqrt[5]{17} = 1.76$  and  $x^5 = 17$ ,  $\therefore x = 1.76$  are both M0 A0 A0

Answer only 1.76: full marks ( A1 A1)

Answer only to a greater accuracy but which rounds to 1.76: A1 A0

(e.g. 1.760, 1.7603, 1.7604, 1.76037 etc)

Answer only 1.8: A1 A0

Trial and improvement: award marks as for "answer only".

Examples

$x = \log_5 17$ $= 1.76$ Working seen, so scheme applied	M0 A0 A0	$5^{1.76} = 17$ Answer only but clear that $x = 1.76$	A1 A1
$5^{1.8} = 17$ Answer only but clear that $x = 1.8$	A1 A0	$5^{1.76}$	M0 A0 A0
$\log_5 17 = x$ $x = 1.760$	A1 A0	$\log_5 17 = x$ $x = 1.76$	A1A1
$x \log_5 = \log 17$ $x = \frac{1.2304...}{0.69897...}$ $x = 1.76$	A1 A1	$x \ln 5 = \ln 17$ $x = \frac{2.833212...}{1.609437...}$ $x = 1.76$	A1 A1
$x \log_5 = \log 17$ $x = \frac{2.57890}{1.46497}$ $x = 1.83$	A1 A0	$\log_{17} 5 = x$ $x = \frac{\log 5}{\log 17}$ $x = 0.568$	M0 A0 A0
$5^{1.8} = 18.1, 5^{1.75} = 16.7$ $5^{1.761} = 17$	A1 A0	$x = 5^{1.76}$	M0A0A0
$x \log_5 = \log 17$ $x = 1.8$	A1A0	$x = \frac{\log 17}{\log 5}$ $x = 1.8$	M1A1 A0
<b>N.B.</b> $x^5 = 17$ $x = 1.76$	M0A0 A0	$\sqrt[5]{17}$ $= 1.76$	M0A0 A0

[3]

3. (a)  $\log 5^x = \log 8$  or  $x = \log_5 8$

Complete method for finding  $x$ :  $x = \frac{\log 8}{\log 5}$  or  $\frac{\ln 8}{\ln 5}$

$= 1.29$  only

A1 3

(b) Combining two logs:  $\log_2 \frac{(x+1)}{x}$  or  $\log_2 7x$

Forming equation in  $x$  (eliminating logs) legitimately

$x = \frac{1}{6}$  or  $0.1\dot{6}$

A1 3

[6]

4. (a)  $\log 3^x = \log 5$   
 $x = \frac{\log 5}{\log 3}$  or  $x \log 3 = \log 5$   
 $= 1.46$

A1

A1 cao 3

(b)  $\log_2\left(\frac{2x+1}{x}\right) = 2$   
 $\frac{2x+1}{x} = 2^2$  or 4  
 $2x+1 = 4x$   
 $x = \frac{1}{2}$  or 0.5

A1 4

[7]

1. Most candidates completed part (a) successfully (sometimes by ‘trial and error’), but sometimes a mark was lost through incorrectly rounding to 3 decimal places instead of 3 significant figures.

Responses to part (b) varied considerably. Many candidates failed to appreciate that  $5^{(2x)}$  is equivalent to  $(5^x)^2$  and either substituted the answer to part (a) into the given equation or took logs of each separate term, resulting in expressions such as  $2x \log 5 - x \log 60 + \log 35 = 0$ . The candidates who managed to form the correct quadratic in  $5^x$  were usually able to proceed to a correct solution, but sometimes the final answers were left as 5 and 7. Notation was sometimes confusing, especially where the substitution  $x = 5^x$  appeared. Some candidates wasted a significant amount of time on part (b), producing a number of different wrong responses with a variety of logarithmic mistakes.

2. The majority of candidates used logarithms in an appropriate way and scored full marks for this question, although some candidates only got as far as  $x = \log_5 17$ . The most common error was to disregard the instruction to give the answer to 3 significant figures. Trial and improvement was seen occasionally. A few candidates incorrectly tried to find  $x$  by writing  $x = \sqrt[5]{17}$ .

### 3. Pure Mathematics P2

Most candidates found this question quite accessible.

- (a) A minority chose to solve this part using trial and improvement. Quite a few did not round to 3 significant figures. Candidates who started with the form  $\log_5 8$  were often not able to progress beyond this.
- (b) This was often started well, but there was some confusion when it came to forming an equation without logarithms. This process often took several more steps than necessary. A small, but significant proportion made no attempt at all. Lines such as “ $\log(x+1) = (\log x)\log(1) = 0$  since  $\log(1) = 0$ ” and “ $\log(x+1) = \log x + \log 1$ ” were popular options amongst the weaker candidates.

### Core Mathematics

Most candidates were able to solve  $5^x = 8$  correctly in part (a), although the answer was sometimes not rounded to 3 significant figures as required. The usual method was to use

$$x = \frac{\log 8}{\log 5},$$

but trial and improvement approaches were sometimes seen.

Part (b) was a more demanding test of understanding of the theory of logarithms. Those who began by using  $\log(x+1) - \log x = \log \frac{x+1}{x}$  were often successful, but common mistakes

such as  $\log(x+1) - \log x = \frac{\log(x+1)}{\log x}$  and  $\log(x+1) = \log x + \log 1$  usually prevented the

candidate from scoring any marks. Confused working, including ‘cancelling’ of logs, was often seen here, and there were many attempts involving an unnecessary change of base.

4. Part (a) was usually answered well although some only got as far as  $x = \log_3 5$  and either couldn’t evaluate it, or failed to read the instruction to give to their answer to 3 significant figures. Some who did evaluate successfully then rounded cumulatively to obtain 1.465 leading to 1.47 but many candidates scored full marks here. A number of candidates used a trial and improvement approach in part (a). Whilst the answer could be obtained in this case it is not a

recommended procedure for this type of question. Part (b) was one of the most discriminating parts of the paper. Whilst a reassuringly high proportion of candidates did achieve full marks here many incorrect procedures were seen.  $\log(2x+1) = \log 2x + \log 1$  was quite common and a number of students drifted from  $\log_2\left(\frac{2x+1}{x}\right)$  to  $\frac{\log_2(2x+1)}{\log_2 x}$  apparently believing them to be equal.